



## Viet's Theorem, Viet's Generalized Theorem

*Ruziyev Jamshid Xudoyberdiyevich*

*Academic Lyceum of Tashkent State University of Economics, teacher of Mathematics*

**Abstract:** *In this article, Viet's theorem is explained in detail and the operations related to this theorem are introduced.*

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The education system is in a constant search for ways to improve the quality of its daily educational work in the classroom, in response to the increasing demand that its graduates reach achievements appropriate for the 21st century. The current mathematics curriculum for middle schools in Israel introduces students mainly to proofs in geometry, and seems unaware of the need for algebraic proofs and the insights rooted in proving theorems. This paper suggests interesting and challenging research activity for 8th and 9th grade students, who are taking their first steps in algebra. This activity would require them to go back in time to the 16th century, and follow the work of François Viète, a French mathematician who was one of the fathers of modern algebra. Algebraic knowledge required to prove Vieta's extended theorem corresponds to the algebraic knowledge of middle school students. The activity suggested in this historic context could be an intellectual experience for the students because it encourages searching for relationships and constancy, understanding the need for proof in algebra, and applying the theorem through solving varied questions and creating new questions. Mathematics teachers who choose this topic and place the students at the center as independent learners, could cultivate and promote mathematical thinking among their students.

Polynomials are to algebra what numbers are to arithmetic. In fact a deep understanding of arithmetic and facility with arithmetic operations paves the way to understanding algebra. Not surprisingly, polynomials problems are quite popular in mathematics competitions. Just as the sets  $\mathbb{Z}$  of integers,  $\mathbb{Q}$  of rational numbers and  $\mathbb{R}$  is real numbers are the underlying sets for algebraic structures, so the polynomials can also be used to build algebraic structures. And just as integers can be represented in many ways so can polynomials. An integer, like 144 can be written in standard form (the ordinary place value form), factored form  $24 \cdot 32$ , or in some hybrid form, like  $(3+9)(24-12)$ . Likewise a polynomial like  $x^2+4x+4$  has standard form  $x^2 + 4x + 4$ , factored form  $(x + 2)(x + 2)$ , and hybrid form  $(x + 2)x + 2(x + 2)$ . The polynomials admit factorizations very much like the factorization of integers into primes, but these factorizations are a bit more complicated. Every real polynomial (one whose coefficients are real numbers) has a complete factorization if we allow complex numbers. For example  $x^2+1 = (x-i)(x+i)$  where  $i$  is the imaginary unit. If we don't allow complex numbers, we must accept parts like  $x^2+1$  as primes. In this case all real polynomials have prime factorizations that include linear factors (of the form  $ax+b$ ) and quadratic factors (with negative discriminant  $D = b^2-4ac$ ). There are just a few theorems that you need to know before attacking the problems below. By far, the most popular theorem about polynomials is Vieta's Theorem.

The following is copied with thanks from The Art of Problem Solving website. Vieta's Formulas were discovered by the French mathematician François Viète. Vieta's Formulas can be used to relate the sum and product of the roots of a polynomial to its coefficients. The simplest application of this is with quadratics. If we have a quadratic  $x^2 + ax + b = 0$  with solutions  $p$  and  $q$ , then we know that we can factor it as:  $x^2 + ax + b = (x - p)(x - q)$  (Note that the first term is  $x^2$ , not  $ax^2$ .) Using the distributive property to expand the right side we now have  $x^2 + ax + b = x^2 - (p + q)x + pq$ . Vieta's Formulas are often used when finding the sum and products of the roots of a quadratic in the form  $ax^2 + bx + c$  with roots  $r_1$  and  $r_2$ . They state that:

$$r_1 + r_2 = -\frac{b}{a}$$

and

$$r_1 \cdot r_2 = \frac{c}{a}.$$

We know that two polynomials are equal if and only if their coefficients are equal, so  $x^2 + ax + b = x^2 - (p + q)x + pq$  means that  $a = -(p + q)$  and  $b = pq$ . In other words, the product of the roots is equal to the constant term, and the sum of the roots is the opposite of the coefficient of the  $x$  term. A similar set of relations for cubics can be found by expanding  $x^3 + ax^2 + bx + c = (x - p)(x - q)(x - r)$ . We can state Vieta's formulas more rigorously and generally. Let  $P(x)$  be a polynomial of degree  $n$ , so  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ , where the coefficient of  $x^i$  is  $a_i$  and  $a_n \neq 0$ . As a consequence of the Fundamental Theorem of Algebra, we can also write  $P(x) = a_n(x - r_1)(x - r_2) \dots (x - r_n)$ , where  $r_i$  are the roots of  $P(x)$ . We thus have that  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = a_n(x - r_1)(x - r_2) \dots (x - r_n)$ . Expanding out the right-hand side gives us  $a_n x^n - a_n(r_1 + r_2 + \dots + r_n)x^{n-1} + a_n(r_1 r_2 + r_1 r_3 + \dots + r_{n-1} r_n)x^{n-2} + \dots + (-1)^n a_n r_1 r_2 \dots r_n$ . The coefficient of  $x^k$  in this expression will be the  $(n - k)$ -th elementary symmetric sum of the  $r$ . Developing mathematical thinking is one of the main goals of the outline of the mathematics curriculum in the scientific track (5 units). Mathematical thinking includes logical thinking, scientific and critical thinking, algorithmic thinking, and understanding basic concepts such as definition, argument, theorem, reverse theorem, and, obviously, proof (as opposed to explanations or examples). Developing mathematical thinking is a process acquired through active learning and experience over years. Research has shown that even students in academic institutions find it difficult to understand formal mathematical texts and to write precise proof (Bills & Tall, 1998). The main experience of high school students in Israel in deductive logical thinking is in geometry. In algebra, algebraic manipulations are not considered conducive of mathematical thinking, but rather to practice technical skills. Tall (1998) claimed that an algebraic argument, which is a formal version of an arithmetic phenomenon (constancy), could be a fitting basis for developing mathematical thinking if it is eventually proven. Cognitive theories argue that significant learning occurs when the student is an active participant in structuring the knowledge in a social interaction that provides meaning. Therefore, a student's reflective thinking about the meaning of the proof and its structure is a crucial component of meaningful learning.

François Viète was one of the great mathematicians of the 16th century. Viète was born at Fontenay-le-Comte in the south of France in 1540. He was a lawyer by training but spent most of his time on mathematics and astronomy. His main work was published in "Introduction to the Art of Analysis", also known as "New Algebra", in 1591. Viète's main idea was to make algebra a powerful mathematical tool, and he was the first to use letters as parameters in equations to express algebraic ideas and generalizations. Among other topics, he studied homogenous equations, and is known to future generations as the man who formulated and proved Vieta's formula.

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