



## Problems of Depicting Spatial Figures on a Plane

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**Abstract:** *The interrelated study of the properties of plane and spatial figures is a means of solving the problem associated with the artificial division of school geometry into planimetry and stereometry. The psychological characteristics of middle school students indicate that the generally accepted practice of forming two-dimensional planar images of earlier three-dimensional spatial images disrupts the natural course of development of students' ideas about space. It turns out to be quite difficult to correct the firmly fixed methods of operating mainly with planar two-dimensional images in the process of studying stereometry*

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Geometry does not exist as a separate discipline in the education system at a technical college. Geometric sections constitute only a small part of the program of the discipline "Mathematics". Nevertheless, it is precisely geometric knowledge and skills, geometric culture and development that are professionally significant today for many modern specialties. Geometric education is the foundation that forms in a person the desire for a visual interpretation of complex phenomena, educates the researcher in the field of his professional knowledge. When constructing images of spatial figures, the Polke-Schwarz theorem plays an important role.

Let an affine frame  $R' = \{A', B', C', D'\}$  be given in the space. An affine frame in space is an ordered quadruple of points in general position, i.e., not lying in the same plane, and no three of them lie on one straight line, or  $R' = \{A', \overrightarrow{A'B'}, \overrightarrow{A'C'}, \overrightarrow{A'D'}\}$ . How to set the image of the affine frame  $R'$  on the image plane  $\Sigma$ ? The answer to this question is given by the Polke-Schwarz theorem: Any ordered quadruple of points  $C \{A, B, C, D\}$  in general position, taken on the plane, can serve as an image of an affine frame congruent or similar to the frame  $R'$ .

As a consequence, the following proposition follows from the Polke-Schwarz theorem: Any quadrangle ABCD together with its diagonals can serve as an image of a tetrahedron in a parallel projection. The image of the pyramid in parallel projection.

Using these two sentences, you can build an image of any n-sided pyramid.

1. We build the image of the polygon, which is the base of the pyramid.
2. Choose an arbitrary point, which is the image of the top of the pyramid (according to the Polke-Schwarz theorem) and build the image of the side edges.
3. If the position of the height is determined by the condition of the problem, then first we build an image of the line on which the height of the pyramid is located, and then we arbitrarily select a vertex on it.

Example. Construct an image of a regular quadrangular pyramid.

1. First, we build a parallelogram ABCD - an image of a square base.
2. Build point O – the image from the center of the base of the pyramid.
3. Then the vertical segment OS is the image of the height, the point S is chosen arbitrarily by the Polke – Schwarz theorem.

For clarity of the image, it is better to take two parallel sides of a parallelogram parallel to the edge of a sheet of paper or the edge of a blackboard (drawing).

Comment.

Images of truncated pyramids often contain errors, the source of which may be reasoning such as the following. The bases of a regular truncated pyramid are squares. Therefore, we will depict them with arbitrary parallelograms, and then connect the tops of the bases. It is easy to see that the resulting image is generally incorrect. To verify this, it is enough to continue the side edges on the image. It turns out that they do not all pass through one point, which is unacceptable for a truncated pyramid.

It is clear that in order to avoid such errors, one must start with the image of the corresponding complete pyramid, and then get the upper base, as a section of this last one drawn in a certain way.

Image of a prism.

1. First, we draw an image of the polygon, which is the base of the prism, according to the rules of parallel design.
2. Through the vertices of this polygon, draw parallel straight lines perpendicular to the edge of the image plane (if the prism is a straight line), lay equal segments on them and connect the ends of the segments. Here we use the fact that parallel design preserves the ratio of segments lying on parallel lines.

Example.

Construct an image of a parallelepiped.

1. We begin the image by constructing an image of the parallelogram base.
2. Then arbitrarily select one of the vertices of the upper base, for example, point A.
3. With the help of parallel transfer of points, we construct points B, C, D on the vector. That is, as described above, through the points, draw straight lines, parallel and put on them the segments  $B_1B = C_1C = D_1D = A_1A$ .

Image of a cylinder.

We will consider a straight circular cylinder. When depicting a cylinder, the projection plane 2 is selected parallel to the cylinder axis, the projection direction is selected parallel to the plane passing through the cylinder axis and perpendicular to the  $\Sigma$  plane. Then the construction of the image of the cylinder can be done as follows:

1. The base of the cylinder (for example, the bottom one) is depicted by an ellipse Q, which can be constructed according to its two mutually perpendicular diameters AB and CD.  $D_1$
2. From the center O of the ellipse in the direction of the minor axis CD, we postpone the segment  $OO_1$  of arbitrary length and thus define the center of the upper base of the cylinder on the image.

3. We build the contour generatrices of the cylinder, that is, the image of the generatrices, which in the original separate the visible part of the cylinder surface from the invisible one. The contour generators are tangent to the ellipse Q, representing the lower base of the cylinder, at points A and B (ends of the major axis of the ellipse) and parallel to CD (minor axis of the ellipse) according to the rule of constructing a tangent to the ellipse at this point.

The image of the ball.

F' – ball (original). If it is projected onto the plane not orthogonally, then the boundary of the projection of the ball will be the ellipse Q. The boundary of the projection of the ball is called the outline of the ball. This image of the ball is not descriptive. If the projection is orthogonal, then the outline of the ball is a circle Q of the same radius as the radius of the ball, and the image of the ball will be a circle. Such an image of the ball is correct and descriptive and is preserved with such a transformation of the plane. In pedagogical practice, a ball is depicted in an orthogonal projection, which means that in this drawing other figures are depicted in an orthogonal projection.

To make the image of the ball visual, they draw, in addition to its outline (circle), some other large circle of the ball (equator), as well as the points of intersection of the diameter of the ball perpendicular to the plane of the equator with the surface of the ball (poles corresponding to the equator). We take the equatorial plane not perpendicular to the plane of images 2, otherwise the equator (the circumference of the great circle) will be represented by a segment, and the poles will be on the outline of the ball.

So, the plane of the equator is not perpendicular to the plane of images 2, so the image of the equator will be an ellipse O, whose major axis [AB] is equal to the diameter of the ball, and the minor axis.

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